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A COMPACTION MODEL FOR LIQUID COMPOSITE MOULDING FIBROUS MATERIALS

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SUMMARY: Liquid Composite Moulding (LCM) processes are a family of advanced composite materials manufacturing processes, which includes the Resin Transfer Moulding (RTM), Injection/Compression Moulding (I/CM) and Vacuum Assisted Resin Infusion (VARI) processes. In an LCM process, many important manufacturing parameters depend on the stresses taken up by the fibrous material before, during and after the fluid-filling stage. For example, the tooling forces in an RTM process and the fill-time and part-thickness in a VARI process depend on this fibre stress. Fibrous materials respond to load in a complex manner, exhibiting viscoelastic effects and undergoing permanent deformation. A new framework for the mathematical modeling of these materials is proposed based on thermomechanical arguments. Physical phenomena of the microscale such as fibre bending, fibre-to-fibre friction and the concept of "frozen energy" are incorporated. The framework is demonstrated for the case of a fibrous material undergoing permanent deformations during a compaction/unloading cycle and the results are compared with experiment.

KEYWORDS: compaction model, fibrous materials, permanent deformation, hysteresis

INTRODUCTION

The many Liquid Composite Moulding (LCM) processes include the rigid-mould Resin Transfer Moulding (RTM) and Injection Compression Moulding (I/CM) processes, the RTMLight process, where more flexible moulds are used, and Vacuum Assisted Resin Infusion (VARI), where one side of the mould is a completely flexible bag. One of the common features of these processes is that a fibrous material is compacted under a load transmitted through the mould and a resin is then forced in some way through the compacted material. Knowledge of the response of the fibrous material to load is critical to an understanding of these processes. Amongst other things, it determines the tooling forces required in RTM, the deflection of the more flexible moulds in RTMLight and the thickness variation of the part during VARI.

The motivation for the work reported here is to produce a full-scale predictive computer simulation of a generic LCM process, of acceptable accuracy. To this end, one requires a model of the response of fibrous materials to load. Fibrous materials respond in a complex non-linear viscoelastic manner [1-3] and undergo significant permanent deformations when loaded [4-5]. The response depends on their microstructural architecture [6] and whether they are dry or lubricated with resin [7]. Ideally, a model would be sophisticated enough to account for some of these effects and yet be not overly-complicated as to make its incorporation in a simulation computationally too-expensive.

In 1946, Van Wyk, in a celebrated paper [8], considered a random assembly of fibres. Assuming that the fibres displaced under force according to the elementary beam theory, he derived the power law relation:

$$\sigma_f = A V_f^3 \tag{1}$$

where σ_f is the stress carried by the assembly, V_f is the fibre volume fraction and A is a material parameter related to the stiffness of the fibres. Many other researchers have followed Wan Wyk's methodology, which involves specifying a probability density function giving the probable orientation of fibres spatially, so estimating the number of fibre-to-fibre contact points in an assembly, determining how the number of contacts increases with compaction, and hence relating stress, via the elastic beam theory, to fibre volume fraction. For example, the increasing fibre-alignment with compaction, allowances for different material symmetries and other refinements have been made [9-12]. Much of this modeling work has been carried out by the textile community. In all these studies, the material is treated as purely elastic. One of the few to consider non-elastic effects were Carnaby and Pan [13], who considered also fibre-to-fibre friction. They carried out unidirectional compression simulations and obtained the much-observed hysteresis load/unload cyclic curve. They also made the important observation that whilst the load is reduced to zero upon unloading, the fibrous assembly still contains strain energy which is "locked into" the assembly by virtue of the fibre-to-fibre friction.

The approach described in the previous paragraph is a micromechanical one, where the fibre diameter, fiber rigidity, coefficient of friction between fibres, etc., are specified and used in the model Another micromechanical approach, useful in the study of knitted and woven fabrics, is to analyse the response of individual yarns and tows, or "unit cells" of tows. These models take the precise fabric microstructural geometric details, for example yarn cross-sectional shape, and predict the deformation under load, e.g. [14-16]. A number of very detailed models are now available, e.g. [17]. These models do not incorporate permanent deformation or viscous effects; this would require one to consider a yarn to be an assembly of many thousands of contacting individual fibres moving relative to each other, and would, at present, be very expensive computationally to simulate.

Another approach is to treat fibrous materials as continua and to apply standard continuum mechanic approaches to the problem. This allows one to consider non-elastic effects without the computational expense involved with micromechanical models. There have been a number of models developed using spring/dashpot/friction-block elements, some linear and some non-linear, which have reproduced the observed permanent deformations [18] and viscoelastic effects

[19-24]. Other approaches include using the classical plasticity theory [25] and standard viscoplastic models, for example the Perzyna model used in [26].

In what follows, a new approach to this modeling problem is proposed. It is a general continuum approach but yet allows one to more readily incorporate at least some of the micromechanical aspects. As such, it has the advantage of the continuum models, their simplicity and use as engineering tools, and yet overcomes to some extent their drawback, their paucity of microscale physics.

A GENERAL FRAMEWORK FOR FIBROUS MATERIAL MODELLING

Following the thermomechanical procedure outlined in, for example, [27-28], the rate at which work is done by the external forces, in the case of isothermal deformations, can be expressed as:

$$\boldsymbol{\sigma} : \mathbf{d} = \dot{\Psi} + \Phi , \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress, $\mathbf{d}$ is the rate of deformation, Ψ is the Helmholtz free energy, a measure of the energy stored, and Φ is the dissipation, the rate at which energy is lost (through heat transfer to the surroundings). One is required to specify the forms of these two energy functions. Using Eqn. 2 as a starting point, a theory of rate-independent plasticity has been developed recently, principally for geomechanics applications [29-30]. Apart from ensuring that the laws of thermodynamics are not violated, an advantage of this formulation is that there is no need to specify the various quantities and functions associated with a conventional plasticity model, for example yield function, flow rule, etc. since these all follow directly from knowledge of the energy functions. The goal here is to apply and particularize elements of this formulation to fibrous materials.

The free energy is of the general form $\Psi = \Psi(\mathbf{e}, \boldsymbol{\alpha})$, where $\mathbf{e}$ represents a strain measure, for example the Green-Lagrange strain, and $\boldsymbol{\alpha}$ represents a set of internal variables which describe the dissipative, inelastic, mechanisms in some way. The dissipation is a function of $\dot{\boldsymbol{\alpha}}$, the rate of change of the internal variable(s). For rate-independent permanent deformations, it must be a homogeneous function of degree one in these rates. The free energy and dissipation then act as potentials from which the stresses and strains can be determined through differentiation.

Fibrous Materials

The following two key points are to be made concerning fibrous materials in general:

(i) They have an ability to acquire "frozen energy"; this is the bending energy of fibres which are locked into bent positions during compression; this energy cannot be accessed without a reversal of any *permanent* deformation.

(ii) They are frictional materials, i.e., energy is dissipated through a frictional mechanism. In this way they are similar to soils and unlike, for example, metals, which involve dislocation-movement mechanisms.

Point (i) implies that the free energy can be expressed in the general form:

$$\Psi(\mathbf{e},\boldsymbol{\alpha}) = \Psi_1(\mathbf{e},\boldsymbol{\alpha}) + \hat{H}(\boldsymbol{\alpha}), \tag{3}$$

with the second term here being the frozen energy, a function of any permanent deformations, as described by $\boldsymbol{\alpha}$, but not of the current strain. Note that the underlying physical mechanism by which recoverable energy is stored (the first term Eqn. 3) and frozen energy is stored is the same, through elastic bending of fibres.

Point (ii) implies that the dissipation is of the form:

$$\Phi(\boldsymbol{\sigma},\boldsymbol{\alpha},\dot{\boldsymbol{\alpha}}), \tag{4}$$

that is, it depends explicitly on the stress. This stress dependence of the dissipation will lead in general to non-associated flow rules [31]; conventional metal plasticity with its associated flow-rules are thus not appropriate for fibrous materials.

A SIMPLE ONE-DIMENSIONAL MODEL

As an illustration of the approach, consider the one-dimensional situation with one internal variable α describing a dissipative mechanism. As a first approximation, assume that the material is uncoupled, in the sense that the instantaneous response, with $\dot{\alpha} = 0$, is independent of α. In that case, the free energy must be of the form [32]:

$$\Psi(\varepsilon,\alpha) = \Psi_1(\varepsilon - \alpha) + \hat{H}(\alpha), \tag{5}$$

and the α corresponds to the permanent deformation. Next, introduce a general dissipation function which satisfies rate-independence and produces a frictional response:

$$\Phi = \phi(\sigma)\dot{\alpha}\,. \tag{6}$$

The stresses and strains are given by:

$$\sigma = \frac{d\Psi_1}{d\varepsilon}, \qquad \varepsilon = -\frac{dG_1}{d\sigma} + \alpha\,. \tag{7}$$

It can then be shown that the material remains elastic for $\sigma < d\hat{H}/d\alpha + \phi(\sigma)$ and yields to permanent deformation otherwise. Note that the function $\hat{H}$ depends on the permanent deformation and hence the yield threshold is ever-increasing as for a kinematically hardening material. The incremental response is given by:

$$\dot{\sigma} = K\dot{\varepsilon}, \qquad K = \left[-\frac{d^2 G_1}{d\sigma^2} + \frac{1 - \dfrac{\partial \phi}{\partial \sigma}}{\dfrac{d^2 \hat{H}}{d\alpha^2}} \right]^{-1} . \tag{8}$$

Example

Consider simple power law terms in the free energy:

$$\Psi(\varepsilon, \alpha) = \frac{E}{n+1}(\varepsilon - \alpha)^{n+1} + h\alpha^m \tag{9}$$

so that

$$\sigma = E(\varepsilon - \alpha)^n, \qquad \varepsilon = -\left(\frac{1}{E}\right)^{\frac{1}{n}} \sigma^{\frac{1}{n}} + \alpha \tag{10}$$

With the dissipation of the form $\Phi = f\sigma\dot{\alpha}$, where f is constant playing the role of a friction coefficient, yield occurs when:

$$\sigma = f\sigma + hm\alpha^{m-1} \tag{11}$$

Results

A Chopped Strand Mat (areal mass 450 g/m^2, fibre density 2.58 g/cm^3) was compacted in a testing machine at 0.05mm/min to $V_f = 0.425$ and then unloaded at the same speed. The response is shown by the solid line in Fig. 1. A slow speed was chosen for this experiment so as to reduce the viscoelastic effects, although they were still seen to be present to some extent (through observed creep and stress relaxation). Model results are shown by the dotted line, with material parameters as given in Table 1. The results are reasonably accurate. According to the model, the material yields at a very low stress and hardens continuously with further compaction. It is assumed that the response is purely elastic upon unloading.

Table 1 Material data

E	2.8e7	$h = E/(n+1)$
n	9	$m = n+1$
f	0.05	

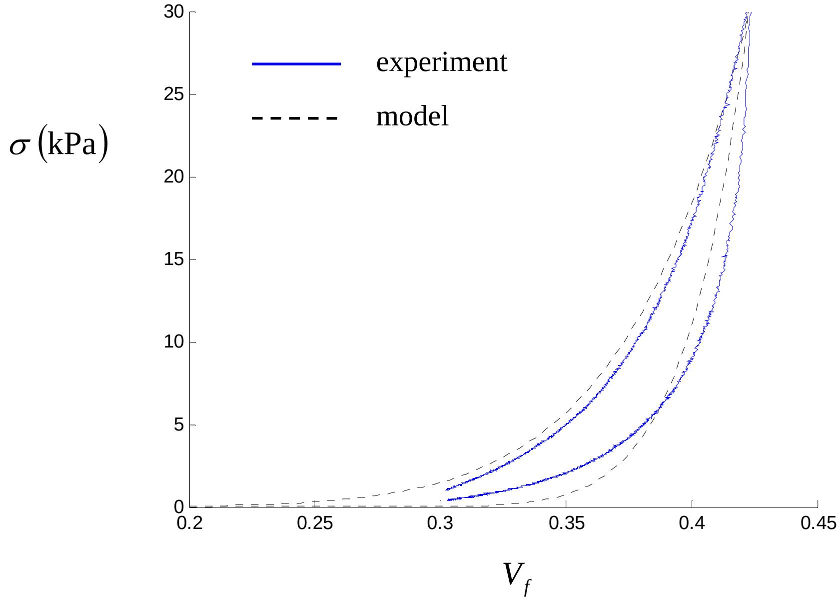

Fig. 1 Results for a cyclic compaction/unloading of CSM.

CONCLUSIONS

A general framework for the modeling of fibrous materials has been presented. The framework relies on information from occurrences on the microscale to suggest particular forms of the model energy functions, and so incorporates key features such as “frozen energy” and frictional dissipation. It has been demonstrated for the case of simple uniaxial compression and unloading, with rate-independent permanent deformations accounted for. The results as shown in Fig. 1 are of acceptable accuracy. It is difficult to compare the efficacy of the model and results with some of the other models of inelastic behaviour mentioned in the introduction, since few if any have the simplicity and reproducibility encompassed in Eqns. 9-11.

Clearly, in order to have a working model for use in LCM simulations, one requires that it exhibits a number of effects which cannot be revealed from a plot of the type shown in Fig. 1. The most important of these are:

- (i) the time-dependent (viscous) response;
- (ii) the de-bulking (preconditioning) response, that is, the change in the stress-strain hysteresis curve with each successive loading cycle;
- (iii) related to (ii), the presence of a maximum possible volume fraction (and a maximum possible permanent deformation) which can be achieved;
- (iv) the different response depending on whether the material is dry or wet.

Also, ideally, a number of further microstructural features of fibrous materials would be accounted for, including the effect of different architectures and of fabric nesting into each other, the so-called “nesting effect”. Further, in order to simulate parts of complex shape, curved or angled, one would require a 3D model incorporating shearing and possibly “cracking” (breakage in tension). All of these present challenges, but this framework seems ideally suited to the task.

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